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Title: FSI modeling, analysis and simulations applied to hemodynamics

Abstract:

Mathematical problems describing the mechanical interaction of a flexible structure with an incompressible fluid flow appear in a wide variety of engineering fields: from the aeroelasticity of sailing boats and parachutes, to sloshing dynamics in tanks, heat exchangers design and micro-encapsulation technology. Fluid-structure interaction (FSI) is particularly ubiquitous in nature. One can think, for instance, of the wings of a bird interacting with the air, the fins of a fish moving through the water, or blood propelled into the arteries. The solid is deformed under the action of the fluid and the fluid flow is disturbed by the moving solid. Such multi-physic phenomena are generally described by heterogeneous systems of non-linear equations with an interface coupling which can be extremely stiff (the so-called added-mass effect). Over the last decade, the development and analysis of efficient numerical methods for these systems has been a very active eld of research. This series of lectures is intended as an introduction to these techniques, with particular emphasis on coupling schemes (stability and accuracy) and partitioned methods delivering efficient simulations of blood flow in large arteries.

Program:

Lecture 1: The non-linear coupled problem

The basic ingredients describing the kinematics of continuum media will be briefly presented. In particular, we will discuss the evaluation of time-derivatives in fields defined in moving domains (arbitrary Lagrangian-Eulerian formalism). Then, we will introduce the equations for the fluid and the solid and the coupling conditions which guarantee a correct energy balance across the interface. We will also discuss how stresses on the interface can be evaluated in order to preserve this energy balance at the discrete level (e.g., using finite elements).

Lecture 2: Stiff Dirichlet-Neumann coupling

In this lecture, the time discretization of the interface coupling conditions will be addressed. We will point out a fundamental issue of incompressible fluid-structure interaction applied to blood flows: the stiffness of Dirichlet-Neumann coupling. Roughly, introducing a small time-lag between the fluid and solid leads to unconditional instability. Then, we will present alternative methods (strongly coupled schemes) which circumvent these issues at the price of solving a computationally demanding coupled system at each time-step. Partitioned algorithms for the solution of these coupled systems will also be presented.

Tutorial 1:

- Partitioned solution strategies: example of implementation (master/slave paradigm with PVM)
- Numerical evidence of the added-mass effect (2D exampled using FreeFem++)

Lecture 3: Projection based and Robin-Neumann splitting schemes

In this lecture, we will show how to circumvent the numerical issues of Lecture 2 using different splitting paradigms. The first approach considers a fractional-step time-marching in the fluid and exploits the resulting velocity/pressure splitting to avoid strong coupling without compromising stability. The second strategy fully uncouples the time-stepping of the fluid and the solid (explicit coupling scheme). The fundamental ingredient here is the notion of interface Robin-Neumann consistency, which will be introduced in the case of the coupling with thin-walled solids. Physiologically relevant simulations of aortic blood blow will be presented using these methods.

Tutorial 2:

- Projection based semi-implicit scheme (2D example using FreeFem++)
- Explicit Robin-Neumann schemes (2D example using FreeFem++)
- Partitioned implementation using independent fluid and solid solvers (master/slave paradigm with PVM)

References:

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3. MF. *Coupling schemes for incompressible fluid-structure interaction: implicit, semi-implicit and explicit*, SeMa J., 55, 59-108, 2011
4. S. Badia, F. Nobile. C. Vergara *Fluid-structure partitioned procedures based on Robin transmission conditions,* J. Comput. Phys. 227(14), 7027-7051, 2008
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The instructor will also provide the pdf of the slides.